PROJECT COMPLETION REPORT

1. Title of the project: A study of k-hyponormality as a bridge between subnormality and hyponormality

2. Principal Investigator: Dr.Munmun Hazarika

3. Implementing Institution: Tezpur University

4. Date of commencement:28.04.2009

5. Planned date of completion: 31.03.2012

6. Actual date of completion: 31.03.2011

7. Objectives as stated in the project proposal:

The success of the Spectral theorem in explaining the structure of normal operators has long motivated the study of several closely related classes of operators, and remarkable progress has been made in the structure theories of subnormal and hyponormal operators. It is well known that normal $\Rightarrow$ subnormal $\Rightarrow$ hyponormal. The converse implications are false, and subsequently the classes of k-hyponormal and weakly k-hyponormal operators have been introduced and studied in an attempt to bridge the gap between subnormality and hyponormality.The classes of k-hyponormal and weakly k-hyponormal Hilbert space operators are intermediate between the important classes of subnormal and hyponormal operators, and as such have received considerable attention.

Objective 1 : To determine the containment relations between the various classes of operators namely subnormal, k-hyponormal, weakly k-hyponormal, polynomially hyponormal, weakly subnormal and so on.

Objective 2 : Following the line of research established for single hyponormal operators, we would like to obtain models for hyponormal n-tuples. One possible such model is by means of singular integrals, where Riesz transforms will probably replace the Hilbert transforms in the single variable case. Another model might be a sub-scalar one, similar to Putinar's model for hyponormal operators.

8. Deviation made from original objectives if any, while implementing the project and reasons thereof: NA

9. Experimental work giving full details of experimental set up, methods adopted, data collected supported by necessary table, charts, diagrams & photographs: NA

10. Detailed analysis of results indicating contributions made towards increasing the state of knowledge in the subject:

We begin our project by undertaking a detailed study of weak 2-hyponormality, also referred to as quadratic hyponormality. For this we further delve into a specific subclass namely, positively quadratic hyponormal operators.

We consider the sequence of positive weights $\alpha(x,y):\sqrt{x},\sqrt{y},\sqrt{\frac{3}{4}},\sqrt{\frac{4}{5}},\cdots$ with a Bergman tail. By using analysis we have shown that there exists an interval (k_1,k_2) about $\frac{2}{3}$ such that if $y\in(k_1,k_2)$ then for $0<x\leq y$, the weighted shift operator $W_{\alpha(x,y)}$ is positively quadratically hyponormal. In fact, using Mathematica graphs we show that the largest such interval is $[k_1,k_2)$, where $k_1\approx 0.630435$ and $k_2\approx 0.737144$ Next, we have shown that the properties of quadratic hyponormality and positive quadratic hyponormality are not preserved under weighted extension and weighted restriction.

Next we begin analyzing hyponormality of Toeplitz operators on the Hardy space. Hyponormality for Toeplitz operators with polynomial symbols can be reduced to classical Schur's algorithm in function theory. We have given the explicit values of the Schur's functions Φ_0, Φ_1, Φ_2 and Φ_3. These values are then used to determine hyponormality condition for trigonometric Toeplitz operators.

The primary question addressed here is: "If φ is a trigonometric polynomial, then when is the trigonometric Toeplitz operator T_φ hyponormal ?" Say we have a trigonometric polynomial φ of the form $\varphi(z)=\sum_{-m}^{N}a_n z^n$ where a_{-m} and a_N are non-zero.It was earlier shown by Kim & Lee that for hyponormality it is necessary to have $m\leq N$ and $|a_{-m}|\leq|a_N|$. Further it was shown that if $|a_{-m}|=|a_N|\neq 0$ then T_φ is hyponormal if and only if the co-efficients of φ satisfy the following symmetry condition: $\overline{a_N}(a_{-1},a_{-2},\ldots,a_{-m}) = a_{-m}(\overline{a}_{N-m+1},\overline{a}_{N-m+2},\ldots,\overline{a}_N)$. We investigate the general situation where $|a_{-m}|<|a_N|$. We consider the situations where the co-efficients a_n of φ satisfy full symmetry everywhere, except at a few finite points.

11. Conclusions summarizing the achievements and indication of scope for future work:

In the project we have generated examples of operators that are positively quadratically hyponormal.Using these we are able to show that the properties of quadratic hyponormality and positive quadratic hyponormality are not preserved under weighted extension and weighted restriction.

In the existing literature, for a trigonometric Toeplitz operator T_φ with $\varphi(z)=\sum_{-N}^{N}a_n z^n$, hyponormality conditions have been framed by considering the condition

$|\bar{a}_N a_{-l} - \bar{a}_{-N} a_l| \leq |a_N|^2 - |a_{-N}|^2$ and $\left(\frac{\bar{a}_N a_{-k} + a_{-N} \bar{a}_k}{\bar{a}_N a_{-l} - a_{-N} \bar{a}_l}\right) = \left(\frac{\overline{a_{N-l+k}}}{a_N}\right)$ for $l \leq N/2$ and $k = 1,\ldots,l-1$. In the present work, we take a different approach and frame alternate hyponormality conditions for trigonometric Toeplitz operator T_φ such that they are independent of the above conditions.

These results and examples will widen the knowledge and the scope of application of the sub-classes of operators namely weakly subnormal, k-hyponormal, weakly k-hyponormal and so on.

We have extensively used Mathematica version 7 and version 5.1 to generate examples to validate new and existing theoretical results. These graphical and numeric illustrations provide deep insights showing the path for new research.

Future Scope : We have studied hyponormality conditions for weighted shift operators and Toeplitz operators in the Hardy space. This work can also be extended to the Bergman space. This will bring out a comparative analysis of the variations in the behavior of these operators when acting on the Bergman space as against the Hardy space.

12. S&T benefits accrued:

i. List of Research publications

Sr.	Authors	Title of paper	Name of the Journal	Volume	Pages	Year
1	M. Hazarika & B. Kalita	A generalized definition of joint hyponormality	Int. J. Math. Anal.	3(36)	1795-1802	2009
2	M. Hazarika & B. Kalita	On positively quadratically hyponormal weighted shifts	Int. J. Cont. Math. Sc.	4(35)	1709-1718	2009
3	M. Hazarika & A. Phukan	On hyponormality of Toeplitz operators with polynomial and symmetric type symbols	Bull. Korean Math. Soc.	48(3)	617-625	2011
4	M. Hazarika & A. Phukan	On hyponormality of trigonometric Toeplitz operators	Int. J. Funct. Anal. Oper. Theory Appl.	3(2)	163-189	2011

ii. Manpower trained on the project : NA

a) Research Scientists or Research Associates
b) No. of Ph.D. produced
c) Other Technical Personnel trained

iii. Patents taken, if any : Nil

iv. Any other outcome. Nil

13. Financial Position:

Sr.	Budget Head	Funds Sanctioned	Expenditure	% of Total cost
1.	Manpower	nil	nil	-
2.	Consumables	Rs.40,000/-	Rs.15,688/-	39.22%
3.	Contingencies	Rs.60,000/-	Rs.13,861/-	23.11%
4.	Travel	Rs.60,000/-	Rs.28,151/-	46.92%
5.	Others, if any	nil	Nil	-
6.	Overhead Expenses	Rs.47,000/-	Rs.30,000/-	63.83%
7.	Equipment	Rs.75,000/-	Rs.74,776/-	99.70%
	Total	**Rs.2,82,000/-**	**Rs.1,62,476/-**	**57.62%**

***Note that only the first installment of Rs.1,50,000/- was received. No further amount was released by DST.

14. Procurement/Usage of Equipment

a)

Sr.	Name of Equipment	Make/ Model	Cost (FE/ Rs.)	Date of Installation	Utilization Rate (%)	Remarks regarding maintenance/ breakdown
1	Laptop & Printer	Dell Vostro1510, HP Laserjet 2015.	Rs. 74,776/-	15.06.2009	100%	No provision for maintenance

b) Plans for utilizing the equipment facilities in future : Typing and printing

Hazarika 31/3/2011

Name and Signature with Date

Dr. Munmun Hazarika

(Principal Investigator)